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A Hierarchical Stability Criterion for Time-Delay Systems based on the Affine Bessel-Legendre Inequality

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Abstract

Recently, the affine Bessel-Legendre(BL) inequality has been presented for a stability analysis of linear systems with time-varying delays. If the degree of the inequality increases, much less conservative stability conditions can be derived but an appropriate design of Lyapunov-Krasovskii functionals and its treatment should be followed. This paper proposes a hierarchical stability criterion of time-delay systems along with the degree of the affine BL inequality based on a novel generalized Lyapunov-Krasovskii functional. With the help of the proposed approach, the stability criteria for various degrees of the inequality can easily be obtained. A numerical example shows the generality and the efficiency of the proposed approach.

요 약

최근 시간 지연 시스템의 안정성 해석을 위해 affine Bessel-Legendre(BL) 부등식이 제안되었다. 제안된 부등식의 차수를 높일수록 더 큰 시간 지연에 대해 안정성을 판별할 수 있는 보다 덜 보수적인 시간 지연 시스템의 안정성 조건을 얻을 수 있지만, 이를 위해서는 차수의 변경에 따른 새로운 Lyapunov-Krasovskii functional의 설계와 이를 처리하기 위한 기법이 요구된다. 본 논문에서는 affine BL 부등식의 차수에 따른 일반화 된 Lyapunov-Krasovskii functional과, 이에 기반한 일반화 된 시간 지연 시스템의 안정성 조건을 제안한다. 제안하는 방법을 활용하면 다양한 부등식의 차수에 따른 안정성 조건을 보다 쉽게 구할 수 있다. 수치 예제의 결과를 통해 제안하는 방법의 일반성과 효율성을 입증한다.

Keywords

time-delay systems, stability analysis, integral inequality, lyapunov-krasovkii functionals, linear matrix inequalities

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Ⅰ. Introduction

Time-delays exist in various practical systems such as mechanical systems, remote control systems, cyber-physical systems, and networked control systems, and they often lead to the degradation of performance, oscillation or even instability of the system[1]. Therefore, the stability analysis of systems with time delays has attracted a lot of attention and numerous researches have been investigated in the past few decades[2]-[14].

When deriving the negativity condition of the time-derivative of Lyapunov-Krasovskii functionals for stability anlaysis of delayed systems, it is important to estimate bounds of the integral terms therein using integral inequalities. To solve the problem, various integral inequalities have been provided in the literature such as Jensen's inequality[2], Wirtinger-based inequality[3], auxiliary function based integral inequalities[4], Bessel-Legendre(BL) inequality[5], free-matrix-based integral inequalities[6]-[8], and affine BL inequality[9]. Especially, in [9], effective stability criteria have been obtained via the affine BL inequality with the certain degrees($N=1,2$). It is mentioned that if the degree N of the affine BL inequality increases, much more precise bounds of the quadratic integral function can be obtained, but to achieve this, the appropriate Lyapunov-Krasovskii functionals should be designed along with the degree N[9]. On the other hand, in [10], the design method of Lyapunov-Krasovskii functionals with an arbitrary degree N of the new integral inequality has been proposed. By utilizing the proposed Lyapunov-Krasovskii functional, hierarchical stability conditions of delayed systems have been obtained.

In this paper, with the help of the idea of [10], a new generalized Lyapunov-Krasovskii functional with an arbitrary degree N of the affine BL inequality is proposed. Furthermore, hierarchical stability conditions along with the degree N is derived via the proposed Lyapunov-Krasovkii functional. By utilizing the new hierarchical stability conditions, the stability criteria for various degrees of the affine BL inequality can easily be obtained. It is also worth noting that the proposed approach can be applied to derive hierarchical stability criteria based on not only the affine BL inequality but recently proposed integral inequalities[7][8]. A numerical example verifies the generality and the efficiency of the proposed stability conditions.

Notations. Throughout the paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. I_n, 0_n, and $0_{m\times n}$ mean $n\times n$ identity matrix, $n\times n$ zero matrix and $m\times n$ zero matrix, respectively. $[X]_{m\times n}$ denotes that X is a $m\times n$ matrix with its (i,j)-th component X_{ij}. The notation $He(A)$ and $col\{x_1,x_2,\cdots,x_n\}$ stand for $A+A^T$ and $[x_1^T,x_2^T,\cdots,x_n^T]^T$, respectively. $X>0(X\geq 0)$ represents that X is a real symmetric positive definite(positive semidefinite) matrix, and $\otimes$ represents a Kronecker product.

Ⅱ. Problem Formulation and Preliminaries

The following linear system with a time-varying delay is considered:

$$\begin{aligned} \dot{x}(t) &= Ax(t)+A_dx(t-h(t)), \\ x(t) &= \phi(t),\ t\in[-h,0] \end{aligned} \tag{1}$$

where $0\leq h(t)\leq h$, $\mu_1\leq \dot{h}(t)\leq \mu_2<1$, $\overline{h}(t)=1-\dot{h}(t)$ and $\phi(t)$ is the initial condition.

To end this section, we quote two lemmas needed to acquire our main results.

Lemma 1 [9]. Let $x(s)\in\mathbb{R}^n$ be a continuously differentiable function for $s\in[a,b]$. For a positive definite matrix $R=R^T>0$, any matrix X, and any integer $N\geq 0$, the following inequality holds:

$$-\int_a^b \dot{x}^T(s)R\dot{x}(s)ds \le -\xi_N^T(t)\Theta(X)\xi_N(t) \qquad (2)$$

where

$$\Theta(X) = XH_N + H_N^T X^T - (b-a)X\overline{R}X^T,$$

$$H_N = \left[\Gamma_N^T(0)\ \Gamma_N^T(1)\ \cdots\ \Gamma_{\mathrm{N}}^{\mathrm{T}}(\mathrm{N})\right]^{\mathrm{T}},$$

$$\overline{R} = diag\left(R^{-1},\ \frac{1}{3}R^{-1},\ \cdots,\ \frac{1}{2N+1}R^{-1}\right),$$

$$\xi_N = \left[x^T(b)\ x^T(a)\ \frac{1}{b-a}\Omega_0^T\ \cdots\ \frac{1}{\mathrm{b-a}}\Omega_{\mathrm{N-1}}^{\mathrm{T}}\right]^{\mathrm{T}},$$

$$\Gamma_N(k) = \left[I\ \ (-1)^{k+1}I\ \ \gamma_{Nk}^0 I\ \cdots\ \gamma_{Nk}^{N-1}I\right],$$

$$\Omega_k = \int_a^b L_k(s)x(s)ds,$$

$$\gamma_{Nk}^i = \begin{cases} -(2i+1)(1-(-1)^{k+i}), & \text{if } i \le k, \\ 0, & \text{if } i \ge k+1, \end{cases}$$

$$L_k(u) = (-1)^k \sum_{l=0}^k \left[(-1)^l \binom{k}{l}\binom{k+l}{l}\right]\left(\frac{u-a}{b-a}\right)^l$$

Lemma 2 [10]. For any integer $i \ge 0$, let x be an integrable function in $[a,b] \to \mathbb{R}^n$. Then, we have

$$I_i(a,b) = \frac{i+1}{b-a}\int_a^b \left(\frac{r-a}{b-a}\right)^i x(r)dr$$
$$= \frac{(i+1)!}{(b-a)^{i+1}}\int_{r_0}^b\int_{r_1}^b \cdots \int_{r_i}^b x(r_{i+1})dr_{i+1}\cdots dr_1$$

where $r_0 = a$.

Remark 1. The affine BL inequality in Lemma 1 has successfully removed the reciprocal convexity, which makes it more difficult to derive the stability criterion, arose from the BL inequality. Note that increasing the degree N can give less conservative stability results, but the Lyapunov-Krasovskii functional also should be newly designed. Fortunately in [9], the method for designing Lyapunov-Krasovskii functional with a given degree N has been described but it is still not easy to derive the new stability condition based on the new Lyapunov-Krasovskii functional whenever the degree N changes, which is the motivation of our work.

III. Proposed Hierarchical Stability Criterion

This section proposes a hierarchical stability criterion of time-delay systems along with the degree of the affine BL inequality (2) based on a new generalized Lyapunov-Krasovskii functional. The following theorem is obtained by utilizing Lemmas 1 and 2.

Theorem 1. For given scalars $h > 0$ and $\mu_1 < \mu_2 < 1$, the system (1) is asymptotically stable if there exist positive definite matrices $[P]_{(3+2N)n\times(3+2N)n}$, and $[Q]_{4n\times 4n}$, $[S]_{4n\times 4n}$, $[R]_{n\times n}$, and any matrices $[X_1]_{(2+N)n\times(1+N)n}$, and $[X_2]_{(2+N)n\times(1+N)n}$ satisfying the following condition for $i,j=1,2$:

$$\begin{bmatrix} \Phi(h,\mu_i) - \Pi_N^{1T}(X_1H_N + H_N^T X_1^T)\Pi_N^1 & \\ -\Pi_N^{2T}(X_2H_N + H_N^T X_2^T)\Pi_N^2 & h\Pi_N^{jT}X_j \\ hX_j^T\Pi_N^j & -hR_N \end{bmatrix} < 0 \quad (3)$$

where

$$\Phi(h(t),\dot{h}(t)) = He\left(G_0^T(\dot{h}(t))PG_1(h(t))\right) + G_2^TQG_2$$
$$-\overline{h}(t)G_3^T(Q-S)G_3 - G_4^TSG_4 + he_0^TRe_0 + \Omega(h(t)),$$

$$\Pi_N^1 = col\left\{e_1, e_2, (C_N \otimes I_N)E_N^1\right\},$$

$$\Pi_N^2 = col\left\{e_2, e_3, (C_N \otimes I_N)E_N^2\right\},$$

$$C_N = \begin{bmatrix} c_0^0 & 0 & \cdots & 0 \\ c_0^1 & c_1^1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ c_0^{N-1} & c_1^{N-1} & \cdots & c_{N-1}^{N-1} \end{bmatrix},$$

$$c_l^k = \frac{(-1)^{l+k}\binom{k}{l}\binom{k+l}{l}}{l+1},$$

$E_N^1 = col\{e_6, e_7, \cdots, e_{6+(N-1)}\}$,

$E_N^2 = col\{e_{6+N}, e_{6+(N+1)}, \cdots, e_{6+(2N-1)}\}$,

$R_N = diag\{R, 3R, \cdots, (1+2N)R\}$,

$G_0(\dot{h}(t)) = col\{e_0, \overline{h}(t)e_4, e_5, e_1 - \overline{h}(t)e_2,$

$G_{0N}^1(\dot{h}(t)), (\overline{h}(t)e_2 - e_3), G_{0N}^2(\dot{h}(t))\}$,

$G_{0N}^1(\dot{h}(t)) = col\{2(e_1 - \overline{h}(t)e_6) - e_7, \cdots$

$l(e_1 - \overline{h}(t)e_{4+l}) - (l-1)e_{5+l}\}, \ l \in [2, N]$,

$G_{0N}^2(\dot{h}(t)) = col\{2(\overline{h}(t)e_2 - e_{6+N}) - e_{7+N}, \cdots$

$l(\overline{h}(t)e_2 - e_{4+N+l}) - (l-1)e_{5+N+l}\}, \ l \in [2, N]$,

$G_1(h(t)) = col\{e_1, e_2, e_3, h(t)G_{1N}^1, (h - h(t))G_{1N}^2\}$,

$G_{1N}^1 = col\{e_6, e_7, \cdots, e_{6+l}\}, \ l \in [0, N-1]$,

$G_{1N}^2 = col\{e_{6+N}, e_{7+N}, \cdots, e_{6+N+l}\}, \ l \in [0, N-1]$,

$G_2 = col\{e_1, e_0, 0_{n \times (5+2N)n}, (e_1 - e_3)\}$,

$G_3 = col\{e_2, e_4, (e_1 - e_2), (e_2 - e_3)\}$,

$G_4 = col\{e_3, e_5, (e_1 - e_3), 0_{n \times (5+2N)n}\}$,

$\Omega(h(t)) = He(h(t)e_6^T(Q_{13}e_0 - Q_{14}e_5)$

$+ (e_1 - e_2)^T(Q_{23}e_0 - Q_{24}e_5)$

$+ h(t)(e_1 - e_6)^T(Q_{33}e_0 - Q_{34}e_5)$

$+ h(t)(e_6 - e_{3)}^{T^T}(Q_{43}e_0 - Q_{44}e_5)$

$+ (h - h(t))e_{6+N}^T(S_{13}e_0 - S_{14}e_5)$

$+ (e_2 - e_3)^T(S_{23}e_0 - S_{24}e_5)$

$+ (h - h(t))(e_1 - e_{6+N})^T(S_{33}e_0 - S_{34}e_5)$

$+ (h - h(t))(e_{6+N} - e_3)^T(S_{43}e_0 - S_{44}e_5))$,

$h_1 = 0, \ h_2 = h$,

$e_0 = Ae_1 + A_d e_{2,}$

and $e_l \in \mathbb{R}^{n \times (5+2N)n}$ for an positive integer $l \in [1, 5+2N]$ are elementary matrices, for example $e_3 = [0_{n \times 2n} \ I_n \ 0_{n \times (2+2N)n}]$

Proof. Consider the generalized Lyapunov-Krasovskii functional with the arbitrary degree N of the inequality in Lemma 1 such that

$$V(t) = \eta_{1N}^T(t)P\eta_{1N}(t) + \int_{t-h(t)}^{t} \eta_2^T(s)Q\eta_2(s)ds \quad (4)$$

$$+ \int_{t-h}^{t-h(t)} \eta_2^T(s)S\eta_2(s)ds$$

$$+ \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta$$

where

$\eta_{1N}(t) = col\{x(t), x(t-h(t)), x(t-h),$

$h(t)\mathbb{I}_N(t-h(t), t), \ (h-h(t))\mathbb{I}_N(t-h, t-h(t))\}$,

$\eta_2(s) =$

The time derivative of (4) can be computed as follows:

$$\dot{V}(t) = \xi^T(t)\Phi(h(t), \dot{h}(t))\xi(t) - \int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds$$

where

$\xi(t) = col\{x(t), x(t-h(t)), x(t-h), \dot{x}(t-h(t)),$

$\dot{x}(t-h), \mathbb{I}_N(t-h(t), t), \mathbb{I}_N(t-h, t-h(t))\}$.

After dividing the range $[t-h, t]$ of the integral term in $\dot{V}(t)$ into two ranges $[t-h, t-h(t)]$ and $[t-h(t), t]$, applying Lemma 1 to the resulting two integral terms gives

$$-\int_{t-h}^{t} \dot{x}^T(s)R\dot{x}(s)ds \le -\xi^T(t)\Psi(h(t))\xi(t) \quad (5)$$

where

$$\Psi(h(t)) = \Pi_N^{1T}(He(X_1 H_N) - h(t) X_1 R_N^{-1} X_1^T)\Pi_N^1$$
$$\Pi_N^{2T}(He(X_2 H_N) - (h-h(t)) X_2 R_N^{-1} X_2^T)\Pi_N^2$$

Combining (5) into $\dot{V}(t)$ yields

$$\dot{V}(t) \le \xi^T(t)\{\Phi(h(t),\dot{h}(t)) - \Psi(h(t))\}\xi(t) \qquad (6)$$

Note that the condition $\Phi(h(t),\dot{h}(t)) - \Psi(h(t)) < 0$ is affine with respect to $h(t)$ and $\dot{h}(t)$, thus if the condition is satisfied at the vertices of the polyhedral set, i.e., $(0,\mu_1),(h,\mu_1),(0,\mu_2),(h,\mu_2)$, the negativity of $\dot{V}(t)$ is ensured for all $(h(t),\dot{h}(t)) \in [0,h] \times [\mu_1,\mu_2]$. Also, by utilizing Schur complement, the negativity condition $\Phi(h(t),\dot{h}(t)) - \Psi(h(t)) < 0$ can be represented as a tractable form of LMIs in (3), which ends the proof.

Remark 2. With the help of the approaches in [10], in Theorem 1, the hierarchical stability criterion is obtained via the generalized Lyapunov-Krasovskii functional in (4) with an arbitrary degree N of the affine BL inequality in Lemma 1. When increasing the degree of the inequality in Lemma 1, $\eta_{1N}(t)$ in the Lyapunov-Krasovskii functional also should be designed appropriately for obtaining a less conservative stability condition. In (4), $\eta_{1N}(t)$ is appropriately designed along with an arbitrary degree N, and from the definition of $I_i(a,b)$ in Lemma 2, its time derivative is calculated by utilizing the following derivation:

$$\frac{d}{dt}\Big(h(t) I_i(t-h(t),t)\Big)$$
$$= (i+1)(x(t) - \overline{h}(t) I_{i-1}) - i\dot{h}(t) I_i,$$
$$\frac{d}{dt}\Big((h-h(t)) I_i(t-h,t-h(t))\Big)$$
$$= (i+1)(\overline{h}(t) x(t-h(t)) - I_{i-1}) + i\dot{h}(t) I_{i,}$$

where $I_{-1}(a,b) = x(a)$

Remark 3. In [9], it is difficult to generalize the stability conditions since it is difficult to find the relation between $\xi_N(t)$ that comes from the inequality in Lemma 1 and the augmented state vector $\xi(t)$ when the degree N of the inequality is changed. It is worth noting that, in Theorem 1, with the help of introducing the matrices Π_N^i such that

$$\Pi_N^i\, \xi(t) = H_N \xi_N^i(t),\ i \in [1,2]$$

where $\xi_N^i(t)$ are $\xi_N(t)$ in Lemma 1 for the intervals $[t-h(t),t]$ and $[t-h,t-h(t)]$, the upper bounds in (5) and (6) can be easily derived as the quadratic form of $\xi(t)$ in (6), which successfully yields the hierarchical stability conditions.

IV. Numerical Example

This section verifies the generality and the efficiency of the proposed result by carrying out the well-known numerical example[2].

Example 1. Consider the delayed system (1) with

$$A = \begin{bmatrix} -2.0 & 0.0 \\ 0.0 & -0.9 \end{bmatrix},\ A_d = \begin{bmatrix} -1.0 & 0.0 \\ -1.0 & -1.0 \end{bmatrix}$$

where $\mu_2 = -\mu_1$. For various μ_1 and μ_2, Table 1 shows the allowable upper bounds of $h(t)$ that guarantee the asymptotic stability of the system and the number of variables(N_v). When the degree N of the inequality increases, the number of variables of Theorem 1 also increases since the dimension of matrices P, X_1 and X_2 increases, and it can be computed by $(4N^2+12N+25)n^2+(N+6)n$.

Table 1. Allowable upper bounds h for different (μ_1,μ_2)

$\mu_2(=-\mu_1)$	0.1	0.5	0.8	N_v
[11]	3.65	2.33	1.93	$3n^2 + 2n$
[12]	4.79	2.68	1.95	$22n^2 + 8n$
[3]	4.70	2.42	2.13	$10n^2 + 3n$
[6]	4.78	3.05	2.61	$65n^2 + 11n$
[13]	4.71	2.60	2.37	$23n^2 + 4n$
[14]	4.83	3.14	2.71	$142n^2 + 18n$
Theorem 1 (N = 1)	4.81	3.10	2.68	$41n^2 + 7n$
Theorem 1 (N = 2)	4.90	3.16	2.73	$65n^2 + 8n$
Theorem 1 (N = 3)	4.90	3.21	2.77	$97n^2 + 9n$
Theorem 1 (N = 4)	4.92	3.22	2.78	$137n^2 + 10n$

From Table 1, it can be seen that Theorem 1 yields larger allowable upper bounds h for most cases compared to stability results in the literature. Only where $N=1$, Theorem 1 is more conservative than the criterion in [14], but it becomes less conservative where $N\geq 2$ both in terms of the performance and the computational complexity. It is worth noting that Theorem 1 successfully yields larger allowable upper bounds h as the degree N of the inequality in Lemma 1 increases, which proves the generality and the effectiveness of Theorem 1.

Remark 4. Note that some recent research also have provided stability criteria with certain degree of the newly proposed inequality such as a generalized free-matrix-based integral inequality(GFMBII)[7] and a generalized integral inequality based on free matrices (GIIBFM)[8]. The proposed approaches in Theorem 1 can be applied to derive hierarchical stability criteria based on those newly proposed integral inequality, which will be our future research direction.

V. Conclusion

This paper proposed the hierarchical stability criterion for linear systems with a time-varying delay based on the affine BL inequality. The generalized Lyapunov-Krasovskii functional with an arbitrary degree N of the inequality was designed, and its time-derivative was successfully dealt with to obtain stability conditions as a tractable form of LMIs. Since it is hierarchical condition, increasing the degree N of the inequality gives much less conservative results, which is clearly verified by the numerical example. It would be an interesting subject for future research to apply the proposed approach to recent results based on the new integral inequalities with few additional number of variables.

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